

# Dynamical Stability of Collapsing Stars in Einstein Gauss-Bonnet Gravity

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## Abstract

This paper is devoted to study the stability/instability of the expansionfree self gravitating source in the framework of Einstein Gauss-Bonnet gravity. The source has been taken as Tolman-Bondi model which is homogenous in nature. The field equations as dynamical equations have been evaluated in Gauss-Bonnet gravity in five dimensions. The junction conditions as well as cavity evaluations equations have been explored in detail. The perturbation scheme of first order has been applied to dynamical as Einstein Gauss-Bonnet field equations. The concept of Newtonian as well post Newtonian approximation have been used to derive general dynamical stability equations. In general this equation represents the stability of the gravitating source. Some particular values of system parameters have been chosen to prove the concept of stability graphically. It has been mentioned that other than chosen the particular values of the parameters the stability of the system will be disturbed , hence it would leads to instability.

**Key Words:** Einstein Gauss-Bonnet Gravity; Gravitational Collapse; Stability of Stars .

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# 1 Introduction

The dynamical instability of the astrophysical objects is the subject of interest in classical physical as well as in general theory of relativity (GR). The motivation of this problem become important when static stellar models are stable against the fluctuations produced by the self gravitational attraction of the massive stars. It is most relevant to structure formation during the different phases of the gravitationally collapsing objects. In the relativistic astrophysics the dynamical stability of the stars was studied by the Chandrasekhar [1] in 1964 , since then a renowned interest has grown in this research area. Herrera et al.[2, 3] have extended the pioneers work for non-adiabatic, anisotropic and viscous fluids. All these investigations imply that adiabatic index  $\Gamma_1$  define the range of instability, for example for the Newtonian perfect fluid such range is  $\Gamma_1 < 4/3$ . Friedman [4] discussed the dynamical instability of neutral fluid sphere in the Newtonian as well as in the relativistic physics and showed that anisotropy enhances the stability if anisotropy is positive throughout matter distribution. Herrera et al. [5] have studied the dynamical instability of the expansion free fluids using perturbation scheme.

Different physical properties of the fluid plays important role in dynamical evaluation of the self-gravitating systems. According to Herrera et al. [6, 7] dissipation terms in the fluid would increase the instability of the collapsing objects. Chan and his collaborators [8]-[11] have showed that anisotropy and radiation would affect the instability range at Newtonian and post-Newtonian approximation. Sharif and Azam [12]-[15] have studied the effects of electromagnetic field on the dynamical stability of the collapsing dissipative and non dissipative fluids in spherical, cylindrical and plane symmetric geometries. This work has been further extended by Sharif and his collaborators [16]-[25] in higher order theories of gravity, like  $f(R)$  and  $f(T)$  and  $f(R, T)$ , in these papers the possible forms of the fluid with electromagnetic field have been discussed in detail.

Till now many quantum theories of gravity have been proposed to investigate the natural phenomenon occurring in astronomy and astrophysics. Among these theories, superstring theory is the most strong candidate which has been extensively investigated for the spacetimes with more than four dimensions. In this theory the effects of extra dimensions becomes more prominent when curvature radius of the central high density regions during gravitational collapse becomes comparable with the curvature radius of the

extra dimensions. From this point of view high density regions can be modeled in a sophisticated way in a theory which deals the extra dimensions. The braneworld universe model which is attractive proposal for the new picture of the universe is based on the superstring theory [26]-[31]. The geometrical interpretation of the braneworld model reveals the fact that we are living on a four dimensional timelike hypersurface which is embedded in more than four dimensional manifold. This suggest that effects of superstring on the formation of back hole during the relativistic gravitational collapse of a star should be investigated explicitly.

The current experiments performed for the tests of inverse square law do not exclude the possibility of the extra dimension even as large as a tenth of millimeter. As observed range of the gravitational force is directly dependent on the size of objects so it is interesting to consider some physical phenomena in the extra dimensions. On the basis of these facts it becomes important to study the general theory of relativity in more than four dimensions. In this regards a class of exact solutions to the Einstein field equations have been determined in the recent years [32]-[36]. These solutions play a significant role in studying the gravitational collapse evolution of the universe. Recently [37]-[45] there has been growing interest to study the higher order gravity, which involves higher order derivatives of curvature terms. One of the most studied extensively higher order gravity theory is the Gauss-Bonnet gravity. This theory is the simplest generalization of general theory of relativity and special case of Lovelock Gravity theory. The Lagrangian of this theory contains just three terms as compared to Lagrangian of Lovelock gravity Theory.

The Gauss-Bonnet gravity theory is used to discuss the nontrivial dynamical systems in the dimensions greater or equal to 5. This theory naturally appears in the low energy effective action of the heterotic string theory. Boulware and Deser [46] formulated the black hole (BH) solutions in  $N$  dimensional gravitational theory with four dimensional Gauss-Bonnet term. These are generalization of  $N$  dimensional solutions investigated by Tangherili [47], Merys and Perry [48]. The spherically symmetric BH solutions and their physical properties have been studied in detail by Wheeler [49]. The structure of topologically nontrivial BHs has been presented by Cai [50]. Kobayashi [51] and Maeda [52] have explored the effects of Gauss-Bonnet term on the structure of Vaidya BH. All these studies show that appearance of the Gauss-Bonnet term in the field Equations would effect the occurrence of BH and Naked singularity during the gravitational collapse. In a

recent paper [53], Jhinag and Ghosh have consider the  $5D$  action with the Gauss-Bonnet terms in Tolman-Bondi model and give an exact model of the gravitational collapse of a inhomogeneous dust. Motivated by these studies, we have discussed the stability of the gravitationally collapsing spheres in Einstein Gauss-Bonnet gravity. This paper is organized as follow: In section **2** the Einstein Gauss-Bonnet field equations and dynamical equations have been presented. The perturbation scheme of first order on the field equations as well as on dynamical equations have been presented in section **3**. Section **4** deals with the Newtonian and post Newtonian approximation and derivation of the stability equation, which is main result of the paper. We summaries the results of the paper in the last section.

## 2 Interior Matter Distribution and Einstein Gauss-Bonnet Field Equations

We begin with the following  $5D$  action:

$$S = \int d^5x \sqrt{-g} \left[ \frac{1}{2k_5^2} (R + \alpha L_{GB}) \right] + S_{matter} \quad (1)$$

where  $R$  is a  $5D$  Ricci scalar and  $k_5^2 = 8\pi G_5$  is  $5D$  gravitational constant. The Gauss-Bonnet Lagrangian is of the form

$$L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \quad (2)$$

where  $\alpha$  is the coupling constant of the Gauss-Bonnet terms. This type of action is derived in the low-energy limit of heterotic superstring theory. In that case,  $\alpha$  is regarded as the inverse string tension and positive definite and we consider only the case with  $\alpha \geq 0$  in this paper. In the  $4D$  space-time, the Gauss-Bonnet terms do not contribute to the Einstein field equations. The action (1) leads to the following set of field equations

$$G_{ab} = G_{ab} + \alpha H_{ab} = T_{ab}, \quad (3)$$

where

$$G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R \quad (4)$$

is the Einstein tensor and

$$H_{ab} = 2 \left[ RR_{ab} - 2R_{a\alpha}R_b^\alpha - 2R^{\alpha\beta}R_{a\alpha b\beta} + R_a^{\alpha\beta\gamma}R_{b\alpha\beta\gamma} \right] - \frac{1}{2}g_{ab}L_{GB}, \quad (5)$$

is the Lanczos tensor.

A spacelike 4D hypersurface  $\Sigma^{(e)}$  is taken such that it divides a 5D space-time into two 5D manifolds,  $M^-$  and  $M^+$ , respectively. The 5D TB space-time is taken as an interior manifold  $M^-$  which represents an interior of a collapsing inhomogeneous and anisotropic sphere is given by [53]

$$ds_-^2 = -dt^2 + A^2 dr^2 + R^2(d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2), \quad (6)$$

where  $A$  and  $R$  are functions of  $t$  and  $r$ . The energy-momentum tensor  $T_{\alpha\beta}^-$  for anisotropic fluid has the form

$$T_{\alpha\beta}^- = (\mu + P_\perp)V_\alpha V_\beta + P_\perp g_{\alpha\beta} + (P_r - P_\perp)\chi_\alpha \chi_\beta, \quad (7)$$

where  $\mu$  is the energy density,  $P_r$  the radial pressure,  $P_\perp$  the tangential pressure,  $V^\alpha$  the four velocity of the fluid and  $\chi_\alpha$  a unit four vector along the radial direction. These quantities satisfy,

$$V^\alpha V_\alpha = -1 \quad , \quad \chi^\alpha \chi_\alpha = 1 \quad , \quad \chi^\alpha V_\alpha = 0 \quad (8)$$

The expansion scalar  $\Theta$  for the fluid is given by

$$\Theta = V^\alpha_{;\alpha}. \quad (9)$$

Since we assumed the metric (6) comoving, then

$$V^\alpha = A^{-1}\delta_0^\alpha \quad , \quad \chi^\alpha = B^{-1}\delta_1^\alpha \quad (10)$$

and for the expansion scalar, we get

$$\Theta = \frac{\dot{A}}{A} + \frac{3\dot{R}}{R}. \quad (11)$$

Hence, Einstein Gauss-Bonnet field equations take the form

$$k_5^2 \mu = \frac{12 \left( R'^2 - A^2 (1 + \dot{R}^2) \right)}{R^3 A^5} \left[ R' A' + A^2 \dot{R} \dot{A} - A R'' \right] \alpha - \frac{3}{A^3 R^2} \left[ A^3 (1 + \dot{R}^2) + A^2 R \dot{R} \dot{A} + R R' A' - A (R R'' + R'^2) \right] \quad (12)$$

$$k_5^2 p_r = -12\alpha \left( \frac{1}{R^3} - \frac{R'^2}{A^2 R^3} + \frac{\dot{R}^2}{R^3} \right) \ddot{R} + 3 \frac{R'^2}{A^2 R^2} - 3 \left( \frac{1 + \dot{R}^2 + R \ddot{R}}{R^2} \right) \quad (13)$$

$$k_5^2 p_\perp = \frac{4\alpha}{A^4 R^2} \left[ -2A \left( A' R' + A^2 \dot{A} \dot{R} - A R'' \right) \ddot{R} + A \left( R'^2 - A^2 (1 + \dot{R}^2) \right) \ddot{A} + 2 \left( \dot{A} R' - A \dot{R}' \right) - \frac{1}{A^3 R^2} \left[ A^3 (1 + \dot{R}^2 + 2R \ddot{R}) + A^2 R (2\dot{R} \dot{A} + R \ddot{A}) + 2R R' A' - 2A (R R'' + R'^2) \right] \right] \quad (14)$$

$$\frac{12\alpha}{A^5 R^3} \left( \dot{A} R' - A \dot{R}' \right) \left( A^2 (1 + \dot{R}^2) - R'^2 \right) - 3 \frac{A \dot{R}' - \dot{A} R'}{A^3 R} = 0 \quad (15)$$

The mass function  $m(t, r)$  analogous to Misner-Sharp mass in  $n$  manifold without  $\Lambda$  is given by [52]

$$m(t, r) = \frac{(n-2)}{2k_n^2} V_{n-2}^k \left[ R^{n-3} (k - g^{ab} R_{,a} R_{,b}) + (n-3)(n-4)\alpha (k - g^{ab} R_{,a} R_{,b})^2 \right], \quad (16)$$

where a comma denotes partial differentiation and  $V_{n-2}^k$  is the surface of  $(n-2)$  dimensional unit space. For  $k=1$ ,  $V_{n-2}^1 = \frac{2\pi^{(n-1)/2}}{\Gamma((n-1)/2)}$ , using this relation with  $n=5$  and Eq.(6), the mass function (16) reduces to

$$m(r, t) = \frac{3}{2} \left[ R^2 \left( 1 - \frac{R'^2}{A^2} + \dot{R}^2 \right) + 2\alpha \left( 1 - \frac{R'^2}{A^2} + \dot{R}^2 \right)^2 \right] \quad (17)$$

The nontrivial components of the Binachi identities,  $T_{;\beta}^{-\alpha\beta} = 0$ , from Eqs.(6) and (7), yield

$$\left[ \dot{\mu} + (\mu + P_r) \frac{\dot{A}}{A} + 3(\mu + P_\perp) \frac{\dot{R}}{R} \right] = 0, \quad (18)$$

and

$$T_{;\beta}^{-\alpha\beta} \chi_\alpha = \frac{1}{A} \left[ P'_r + 3(P_r - P_\perp) \frac{R'}{R} \right] = 0 \quad (19)$$

Using field equations and Eq.(17), we may write

$$m' = \frac{2}{3}k_5^2\mu R'R^3 \quad (20)$$

In the exterior region to  $\Sigma^{(e)}$ , we consider Einstein Gauss-Bonnet Schwarzschild solution which is given by [54]

$$ds_+^2 = -F(\rho)d\nu^2 - 2d\nu d\rho + \rho^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \quad (21)$$

where  $F(\rho) = 1 + \frac{\rho^2}{4\alpha} - \frac{\rho^2}{4\alpha}\sqrt{1 + \frac{16\alpha M}{\pi\rho^4}}$

The smooth matching of the 5D anisotropic fluid sphere (6) to GB Schwarzschild BH solution (21), across the interface at  $r = r_{\Sigma^{(e)}} = \text{constant}$ , demands the continuity of the line elements and extrinsic curvature components (i.e., Darmois matching conditions), implying

$$dt \stackrel{\Sigma^{(e)}}{=} \sqrt{F(\rho)}d\nu, \quad (22)$$

$$R \stackrel{\Sigma^{(e)}}{=} \rho, \quad (23)$$

$$m(r, t) \stackrel{\Sigma^{(e)}}{=} M, \quad (24)$$

$$\begin{aligned} & -12\alpha \left( \frac{1}{R^3} - \frac{R'^2}{A^2 R^3} + \frac{\dot{R}^2}{R^3} \right) \ddot{R} + 3 \frac{R'^2}{A^2 R^2} - 3 \left( \frac{1 + \dot{R}^2 + R\ddot{R}}{R^2} \right) \\ & \stackrel{\Sigma^{(e)}}{=} \frac{12\alpha}{A^5 R^3} \left( \dot{A}R' - A\dot{R}' \right) \left( A^2 \left( 1 + \dot{R}^2 \right) - R'^2 \right) - 3 \frac{A\dot{R}' - \dot{A}R'}{A^3 R} \end{aligned} \quad (25)$$

Comparing Eq.(25) with (13) and (15) (for detail see [13]), we get

$$p_r \stackrel{\Sigma^{(e)}}{=} 0. \quad (26)$$

Hence, the matching of the interior inhomogeneous anisotropic fluid sphere (6) with the exterior vacuum Einstein Gauss-Bonnet spacetime (21) produces Eqs.(6) and (24). These are the necessary and sufficient conditions for the smooth matching of interior and exterior regions of a star on boundary surface  $\Sigma^{(e)}$ .

It is well known that the expansionfree models present an internal vacuum cavity. The boundary surface between the external cavity and interior the

fluid is labeled by  $\Sigma^{(i)}$  then the smooth matching of the Minkowski spacetime within the cavity to the fluid distribution over  $\Sigma^{(i)}$ , yields

$$m(r, t) \stackrel{\Sigma^{(i)}}{=} 0. \quad (27)$$

$$p_r \stackrel{\Sigma^{(i)}}{=} 0. \quad (28)$$

The physical applications of expansionfree models are wide in astrophysics and astronomy. For example, it may help to explore the structure of voids on cosmological scales[55]. By definition Voids are the sponge like structures and occupying 40-50 percent of the entire universe. There are commonly two types of the voids: mini-voids [56] and macro-voids[57] On the basis of Observational data analysis the voids are neither empty nor spherical. For the sake of further exploration about voids they are considered as vacuum spherical cavities around the fluid distribution.

### 3 The Perturbation Scheme

In this section, we introduce the perturbation scheme, for this purpose it is assumed that initially fluid is in static equilibrium implying that the fluid is described by only such quantities which have only radial dependence. Such quantities are denoted by a subscript zero. We further assume as usual, that the metric functions  $A(t, r)$  and  $R(t, r)$  have the same time dependence in their perturbations. Therefore, we consider the metric and material functions in the following form

$$A(t, r) = A_0(r) + \epsilon T(t)a(r), \quad (29)$$

$$R(t, r) = R_0(r) + \epsilon T(t)c(r), \quad (30)$$

$$\mu(t, r) = \mu_0(r) + \epsilon \bar{\mu}(t, r), \quad (31)$$

$$P_r(t, r) = P_{r0}(r) + \epsilon \bar{P}_r(t, r), \quad (32)$$

$$P_\perp(t, r) = P_{\perp 0}(r) + \epsilon \bar{P}_\perp(t, r), \quad (33)$$

$$m(t, r) = m_0(r) + \epsilon \bar{m}(t, r), \quad (34)$$

$$\Theta(t, r) = \epsilon \bar{\Theta}(t, r), \quad (35)$$

where  $0 < \epsilon \ll 1$  and we choose the Schwarzschild coordinates with  $R_0(r) = r$ . Using Eqs.(29)-(33), we have from Eqs.(12)-(15) the following static con-



figuration

$$k\mu_0 = \frac{3}{r^3 A_0^4} \left[ 4\alpha \left( \frac{A'_0}{A_0} - A_0 \right) - r A_0^2 \left( A_0 + \frac{A'_0}{A_0} r - 1 \right) \right], \quad (36)$$

$$kP_{r0} = 3 \left[ \frac{1}{r^2 A_0^2} - \frac{1}{r^2} - 1 \right], \quad (37)$$

$$kP_{\perp 0} = \frac{-1}{A_0^2 r^2} \left[ A_0^2 + 2r \frac{A'_0}{A_0} - 2 \right]. \quad (38)$$

Also from Eqs.(12)-(15), we obtain the following form of the perturbed field equations

$$\begin{aligned} k\bar{\mu} &= \frac{3T}{r^2 A_0^3} \left[ 4\alpha \left( \frac{a'}{r A_0^2} + \frac{3A'_0 c'}{r A_0^2} - \frac{c''}{r A_0} - \frac{a'}{A_0 r} - \frac{c' A'_0}{r A_0} \right. \right. \\ &\quad \left. \left. + \frac{c''}{r} + \frac{3a A'_0}{r A_0^2} - \frac{5a A'_0}{r A_0^2} - \frac{c A'_0}{r^2 A_0^2} + \frac{c A'_0}{r^2 A_0} \right) \right. \\ &\quad \left. - \left( -3a A_0 + 2c - 3ar \frac{A'_0}{A_0} + A'_0 c + ra' \right. \right. \\ &\quad \left. \left. + A'_0 r c' - A_0 r c'' - A_0 + 2a - 2A_0 c' \right) \right] - \frac{2Tc}{r} k\mu_0 \end{aligned} \quad (39)$$

$$k\bar{P}_r = \frac{3\ddot{T}c}{r} \left[ 1 - 4\alpha \left( 1 - \frac{1}{r^2 A_0^2} \right) \right] - \frac{6T}{r^2} \left( \frac{a}{A_0^3} - \frac{c'}{A_0^2} + cr \right) - \frac{2Tc}{r} kP_{r0}, \quad (40)$$

$$\begin{aligned} k_5^2 \bar{P}_\perp &= \frac{\ddot{T}}{A_0^3 r^2} \left[ 4\alpha \left( a(1 - A_0^2) - 2A'_0 c \right) - A_0 r c (A_0^2 + r) \right] \\ &\quad + \frac{8\alpha \dot{T}}{A_0^3 r^2} \left( \frac{a}{A_0} - c' \right) + \frac{T}{A_0^2 r^2} \left[ 2rc' \left( \frac{A'_0}{A_0} \right) + 2a \left( \frac{A'_0}{A_0} \right) - 2rc'' \right. \\ &\quad \left. + 2r \left( \frac{a}{A_0} \right)' - 4c'r - 5 \left( \frac{a}{A_0} \right) - 4ar \left( \frac{A'_0}{A_0} \right) \right] - \frac{2Tc}{r} kP_{\perp 0}, \end{aligned} \quad (41)$$

$$\frac{12\alpha \dot{T}}{A_0^5 r^3} (A_0^2 a - A_0^3 c - a - A_0 c) - \frac{3\dot{T}}{A_0^3 r} (A_0 c' - a) = 0. \quad (42)$$

For the expansion given in Eq.(11), we have

$$\bar{\Theta} = \dot{T} \left( \frac{a}{A_0} + \frac{3c}{R_0} \right). \quad (43)$$

The Binachi identities Eqs.(18) and (19) with (29)-(33), yield the static configuration

$$P'_{r0} + \frac{3}{r} (P_{r0} - P_{\perp 0}) = 0 \quad (44)$$

and for the perturbed configuration

$$\frac{1}{A_0} \left[ \bar{P}'_r + \frac{3}{r} (\bar{P}_r - \bar{P}_\perp) + 3 (P_{r0} - P_{\perp 0}) T \left( \frac{c}{r} \right)' \right] = 0, \quad (45)$$

$$\bar{\mu} = - \left[ (\mu_0 + P_{r0}) \frac{a}{A_0} + \frac{3c}{r} (\mu_0 + P_{\perp 0}) \right] T. \quad (46)$$

The total energy inside  $\Sigma^{(e)}$  up to a radius  $r$  given by Eq.(17) with Eqs.(29),(30) and (34) becomes

$$m_0 = \frac{3}{2} \left[ \left( 1 - \frac{1}{A_0^2} \right) (r^2 + 2\alpha (A_0^2 - 1)) \right], \quad (47)$$

$$\bar{m} = \frac{3T}{A_0^2} \left[ \left( A_0^2 c r - c - c' r^2 + r^2 \frac{a}{A_0} \right) - \frac{\alpha}{A_0^2} (A_0^2 - 1) \left( c' - \frac{a}{A_0} \right) \right] \quad (48)$$

From the matching condition Eq.(26), we have

$$P_{r0} \stackrel{\Sigma^{(e)}}{=} 0, \quad \bar{P}_r \stackrel{\Sigma^{(e)}}{=} 0, \quad (49)$$

For  $c \neq 0$ , which is the case that we want to study, with (40),(42) and (49) we obtain

$$\ddot{T} \beta - \gamma T = 0, \quad (50)$$

where

$$\beta = 1 - 4\alpha \left( 1 - \frac{1}{r^2 A_0^2} \right), \quad \gamma = \frac{2}{rc} \left( \frac{a}{A_0^3} - \frac{c'}{A_0^2} + rc \right)$$

The general solution of Eq.(50) is actually the linear combination of two solutions one of these corresponding to stable (oscillating) system while other corresponds to unstable (non-oscillating) ones. As in the present case, we are interested to establish the range of instability, so we restrict our attention to the non oscillating ones, i.e., we assume that  $a(r)$  and  $c(r)$  attain such values on  $r_{\Sigma^{(e)}}$  that  $\psi_{\Sigma^{(e)}} = \left( \frac{\beta}{\gamma} \right)_{\Sigma^{(e)}} > 0$ . Then

$$T = \exp(-\sqrt{\psi_{\Sigma^{(e)}}} t) \quad (51)$$

representing collapsing sphere as areal radius becomes decreasing function of time.

The dynamical instability of collapsing fluids can be well discussed in term of adiabatic index  $\Gamma_1$ . We relate  $\bar{P}_r$  and  $\bar{\mu}$  for the static spherically symmetric configuration as follows

$$\bar{P}_r = \Gamma_1 \frac{P_{r0}}{\mu_0 + P_{r0}} \bar{\mu}. \quad (52)$$

We consider it constant throughout the fluid distribution or at least, throughout the region that we want to study.

## 4 Newtonian and Post Newtonian Terms and Dynamical Stability

This section deals to identify the Newtonian (N), post Newtonian (pN) and post post Newtonian (ppN) regimes. For this purpose we convert the relativistic units into c.g.s. units and expands all the terms in dynamical equations upto the  $C^{-4}$  ( $C$  being speed of light). In this analysis for the different regimes following approximation will be applicable

- N order: terms of order  $C^0$ ;
- pN order: terms of order  $C^{-2}$ ;
- ppN order: terms of order  $C^{-4}$ .

These terms are analyzed for the stability conditions appearing in the dynamical equation in the N approximations while pN and ppN are neglected. Thus, for N approximation, we assume

$$\mu_0 \gg P_{r0}, \quad \mu_0 \gg P_{\perp 0} \quad (53)$$

For the metric coefficient expanded up to pN approximation, we take

$$A_0 = 1 + \frac{Gm_0}{C^2 r}, \quad (54)$$

where  $G$  is the gravitational constant and  $C$  is the speed of light. With the help of equations obtained in previous sections, we can formulate the

dynamical equation with expansion-free condition which is aim of our study. The key equation for the dynamical equation is Eq.(45).

The expansion-free condition  $\Theta = 0$  implies from (43)

$$\frac{a}{A_0} = -3\frac{c}{r}, \quad (55)$$

with (55), we have for (46) that

$$\bar{\mu} = 3(P_{r0} - P_{\perp 0})T\frac{c}{r}. \quad (56)$$

This equation explains how perturbed energy density of the system originates from the static background anisotropy.

Also, with (52) and (56) we have

$$\bar{P}_r = 3\Gamma_1 \frac{P_{r0}}{\mu_0 + P_{r0}} (P_{r0} - P_{\perp})T\frac{c}{r} \quad (57)$$

From equations (36) and (47), we have

$$\frac{A'_0}{A_0} = \frac{(r + m_0)[(r + m_0)^3 k_5^2 \mu_0 + 12\alpha]}{12\alpha r - 3r(r + m_0)} \quad (58)$$

Next, we develop dynamical equation by substituting Eq.(41) along with Eqs. (55),(54),(50) and (58) in Eq. (45) and using the radial functions  $a(r) = a_0 r$ ,  $c(r) = c_0 r$ , where  $a_0$  and  $c_0$  are constants. After a tedious algebra ( a detail procedure can be followed in [6]), we obtain the dynamical equation at pN order ( with  $c = G = 1$ )

$$\begin{aligned} & \left(12\alpha r - 3r(r + m_0)\right) \left[ 3\psi(r + m_0)^2 \left( 12\alpha c_0 m_0 (m_0 + 2r) - 96\alpha^2 c_0 r^3 (r + m_0) \right. \right. \\ & \left. \left. - c_0 \left( (r + m_0)^2 + r^3 \right) \right) + 8\alpha r^8 \sqrt{\psi} c_0 + 3r^3 c_0 (r + m_0) (4r - 15) + 6r^3 (r + m_0)^3 c_0 k_5^2 P_{\perp 0} \right. \\ & \left. + 3r^3 (r + m_0)^3 k_5^2 (P_{r0} - P_{\perp 0}) \right] + (r + m_0) \left[ 216\alpha r^3 c_0 (1 - 2r)(r + m_0)^2 - 72\alpha r^4 c_0 \right] \\ & = \left[ 24r^3 \alpha \psi c_0 (r + m_0) + 6r^4 c_0 + 18c_0 r^3 (2r - 1)(r + m_0) \right] k_5^2 \lambda \left( r^{n+1} + \frac{2}{3} \left( \frac{r^{n+4}}{n+4} - \frac{r_i^{n+4}}{n+4} \right) \right) \end{aligned} \quad (59)$$

Here, we have used Eq.(20) and considered an energy density profile of the form  $\mu_0 = \lambda r^n$ , where  $\lambda$  is positive constant and  $n$  is also a constant

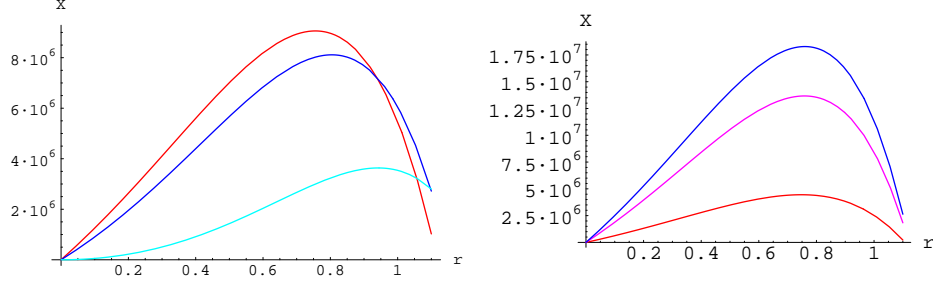


Figure 1: The left graph is plotted for  $\alpha = 1, 2, 2.5$  and right graph is plotted for  $c_0 = -1, -3, -5$ . For both graphs  $m_0 = 10$ ,  $(P_{r0} - P_{\perp 0}) = 5$ ,  $P_{\perp 0} = 10$  are common

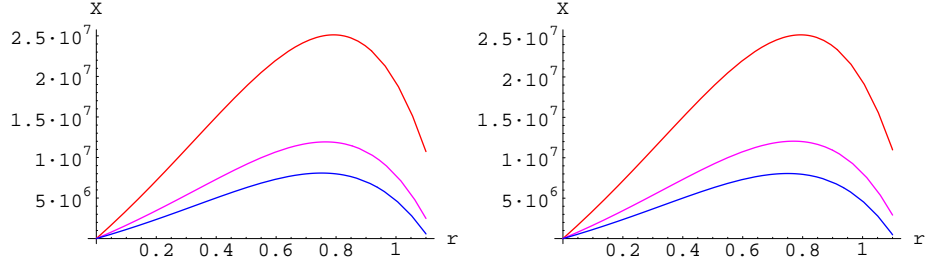


Figure 2: The left graph is plotted for  $m_0 = 9.5, 10.5, 12$  and right graph is plotted for  $(P_{r0} - P_{\perp 0}) = 2, 4, 6$ . For both graphs  $\alpha = 1$ ,  $c_0 = -2$ ,  $P_{\perp 0} = 10$  are common.

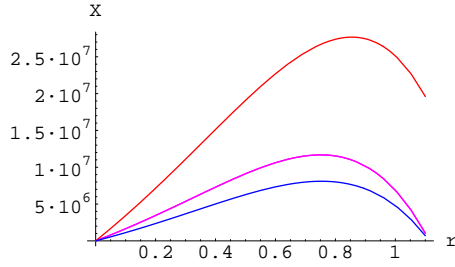


Figure 3: This graph is plotted for  $(P_{r0} - P_{\perp 0}) = 2$ ,  $\alpha = 1$ ,  $c_0 = -2$ ,  $P_{\perp 0} = 10, 12, 13$ .

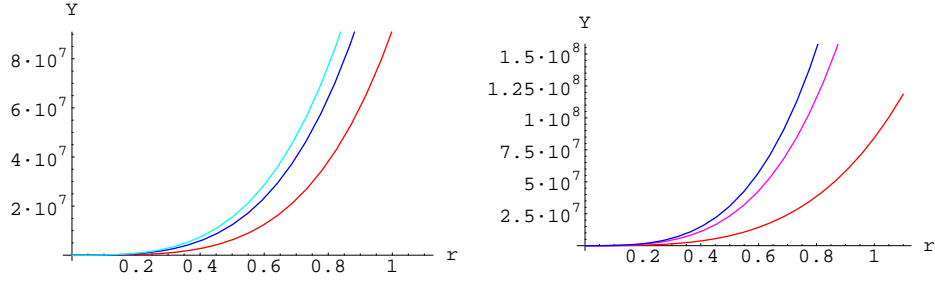


Figure 4: The left graph is plotted for  $\alpha = 1, 2, 2.5$  and right graph is plotted for  $c_0 = -1, -3, -5$ . For both graphs  $m_0 = 10, \lambda = 2, n = 4$  are common

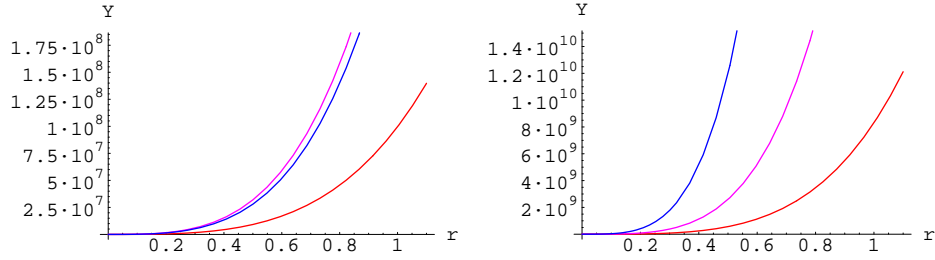


Figure 5: The left graph is plotted for  $r_i = 0.5, 0.7, 0.9$  and right graph is plotted for  $n = 2, 4, 6$ . For both graphs  $\alpha = 1, c_0 = -2, \lambda = 2, m_0 = 10$  are common.

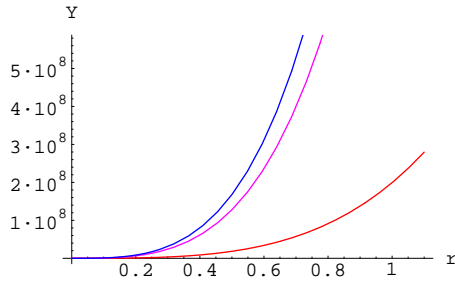


Figure 6: This graph is plotted for  $\alpha = 1, c_0 = -2, n = 2, m_0 = 10, \lambda = 2, 4, 6$

whose value ranges in the interval  $-\infty < n < \infty$ . In order to fulfill the stability of expansion free fluids, we have to prove that both sides of Eq.(59) produces positive results, which is analytically impossible. We represent left side of Eq.(59) as  $X(r)$  and right side of this equation by  $Y(r)$ . We prove graphically that for particular values of the parameters involved in Eq.(59) both  $X(r)$  and  $Y(r)$  positive. The positivity of  $X(r)$  and  $Y(r)$  is shown in figures **(1-3)** and **(4-6)**, respectively. The values of the parameters for which  $X(r)$  and  $Y(r)$  remain positive (system predicts range of stability) are mentioned below each graph and other than these values system becomes unstable.

## 5 Summary

This paper deals with dynamical instability of the expansionfree anisotropic fluid at Newtonian and post Newtonian order in the frame work of Einstein Gauss-Bonnet gravity, which is vast play ground for higher dimensional analysis of general relativity . For a gravitating source which has non zero expansion scalar, the instability range of a self gravitating source can be defined by the adiabatic index  $\Gamma_1$ , which measures the compressibility of the fluid under consideration. On the other hand, for an expansionfree case as we are dealing, the instability explicitly depends upon the energy density, radial pressure, local anisotropy of pressure and Gauss-Bonnet coupling constant  $\alpha$  at Newtonian approximation, but it appears to be independent of the adiabatic index  $\Gamma_1$ . In other words the stiffness of gravitating source at Newtonian and post Newtonian approximation does not play any role for the investigation of the stability of system. We would like to mention that anisotropy in pressure, inhomogeneity in the energy density and Gauss-Bonnet coupling constant  $\alpha$  are the key factors for studying the the structure formation as well as evolution of shearfree anisotropic astrophysical objects.

We have formulated two dynamical equations how gravitating objects evolve with time? and what is the final outcome of such evolution?. One of these dynamical equations is used to separate the terms which have Newtonian and post Newtonian order by using the concept of relativistic and c.g.s units. The post post Newtonian regimes are absent in the present analysis, it not due to Gauss-Bonnet gravity, it seems to occur due to the geodesic properties of the spacetime used in which  $g_{00} = 1$ . This condition is in fact Newtonian limit of general relativity. The second dynamical equation is used

to discuss the instability range of expansionfree fluid upto pN order.

The first order perturbation scheme has been applied on the metric functions and matter variables appearing in the Gauss-Bonnet field equations and dynamical equations. The analysis of resulting dynamical equations shows that stability is independent of adiabatic index  $\Gamma_1$  due to expansion-free fluid. The instability depends on the density profile, local anisotropy, Gauss-Bonnet coupling constant and some other parameters. The instability required that resultant of all term on left side of equation (59) should be positive and equal to resultant of all terms on right side of that equation. It is impossible to show analytically from Eq.(59), so we have proved this result for a particular values of the parameters appearing in Eq.(59). The domain of the parameters is taken conveniently to show both sides positive Fig. (1-6). The parameters have following values for which system satisfies stability conditions:  $1 \leq \alpha \leq 2.5$ ,  $-4 \leq c_0 \leq -1.95$ ,  $9.5 \leq m_0 \leq 12$ ,  $2 \leq (P_{r0} - P_{\perp 0}) \leq 6$ ,  $10 \leq P_{\perp 0} \leq 13$ ,  $0.5 \leq r_i \leq 0.9$ ,  $2 \leq \lambda \leq 15$ ,  $4 \leq n \leq 8$ . We have the novel values of the parameters one can carry actual calculations for the values of the parameters by introducing some restrictions on the system under consideration. This work with electromagnetic and heat flux in the presence of non-geodesic model i.e.,  $g_{00} \neq 1$  is under progress [58].

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